Point location using the Planar Seperator Theorem

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- Oldest method with $O(\log n)$ search cost

  Seperator: SICOMP 1979 Lipton / Tarjan

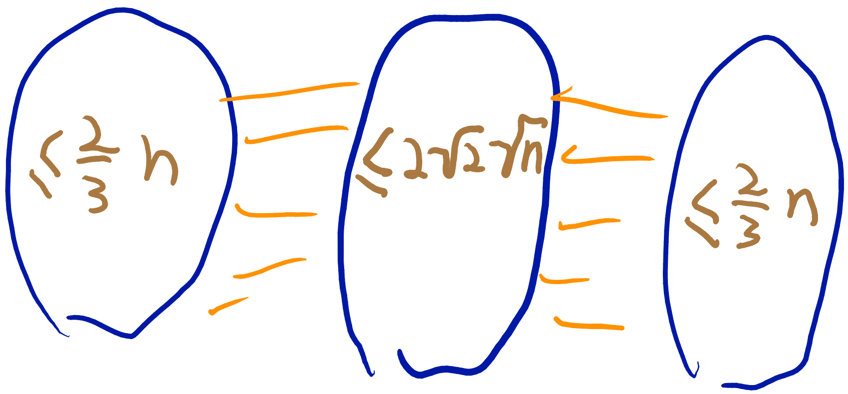
  Using for point location: 1977 Lipton / Tarjan

Planar Seperator Theorem

Every planar graph has a seperator of size

$$2\sqrt{2}\sqrt{n}$$

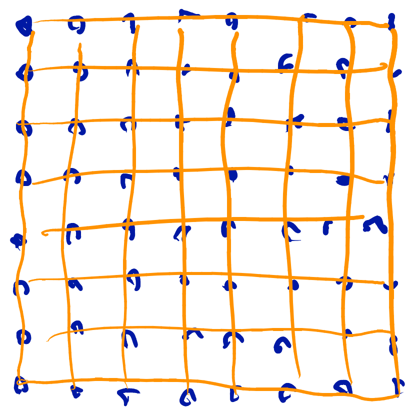
which splits the graph into 2 parts of size $\leq \frac{2}{3}n$



The seperator can be computed in $O(n)$ time.

Planar Seperator Theorem

Lower bound of $\sqrt{n}$



Planar Seperators for Graphs of Bounded Radius

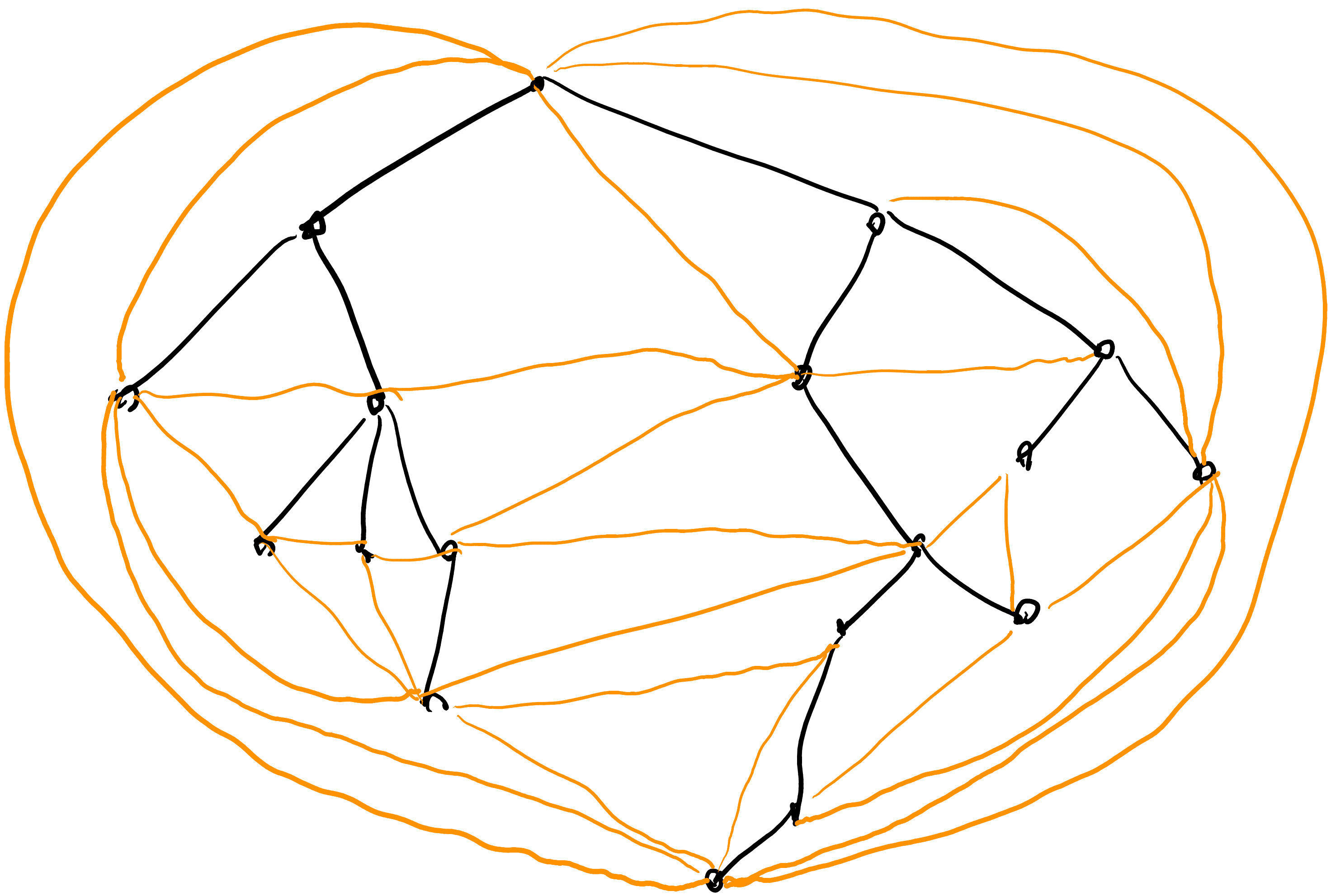
If G has a spanning tree of height r and vertex weights summing to n, there is a seperator of size $2r+1$ such that the two sets have weight $\leq \frac{2}{3}n$

Proof: No vertex has weight $\geq \frac{1}{3}n$ (else it is a seperator)

Triangulate. Each non-tree edge defines a cycle using the tree

Pick the best such edge. By several cases of an exchange argument, it must be a seperator.

Planar Seperators for Graphs of Bounded Radius



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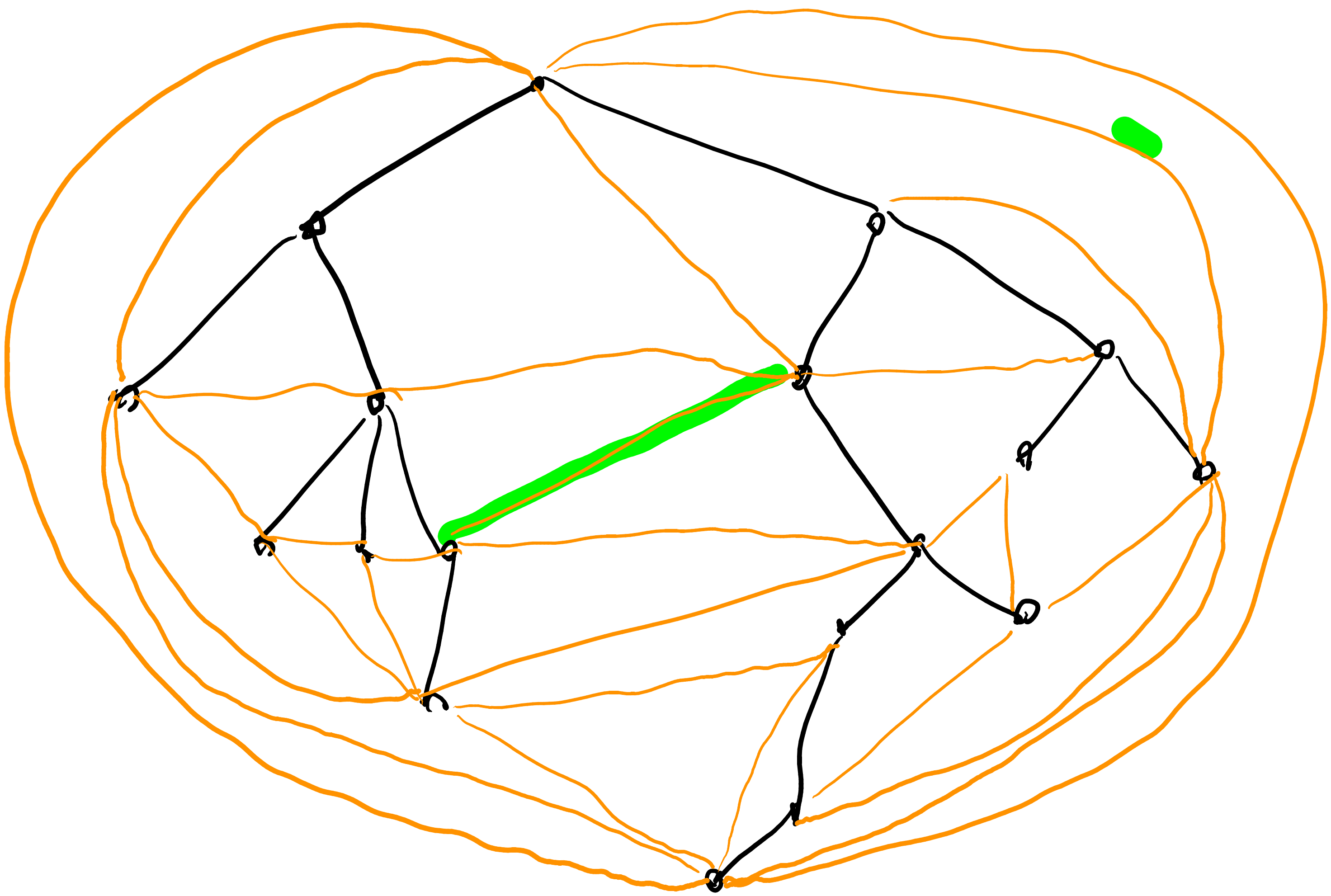

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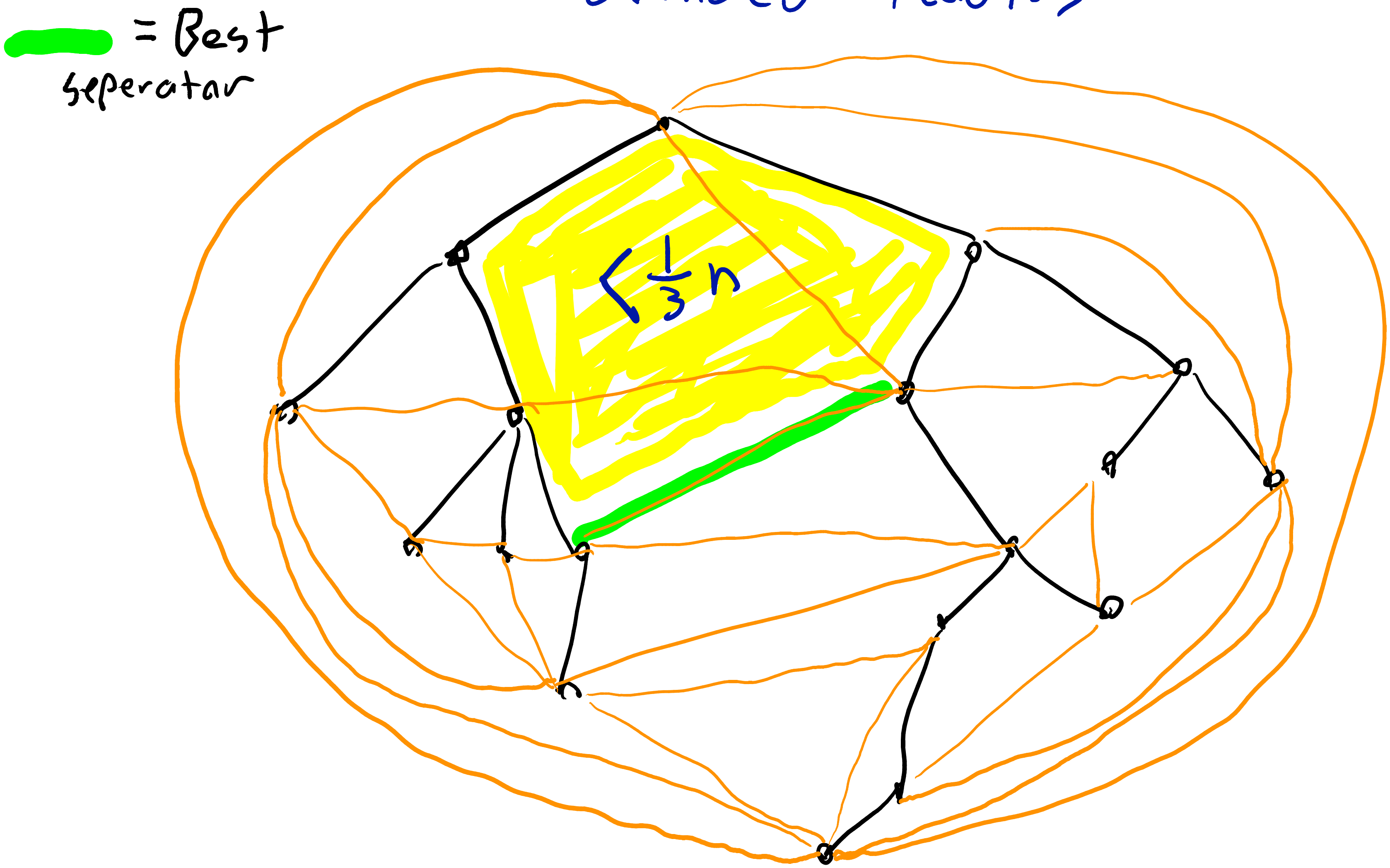
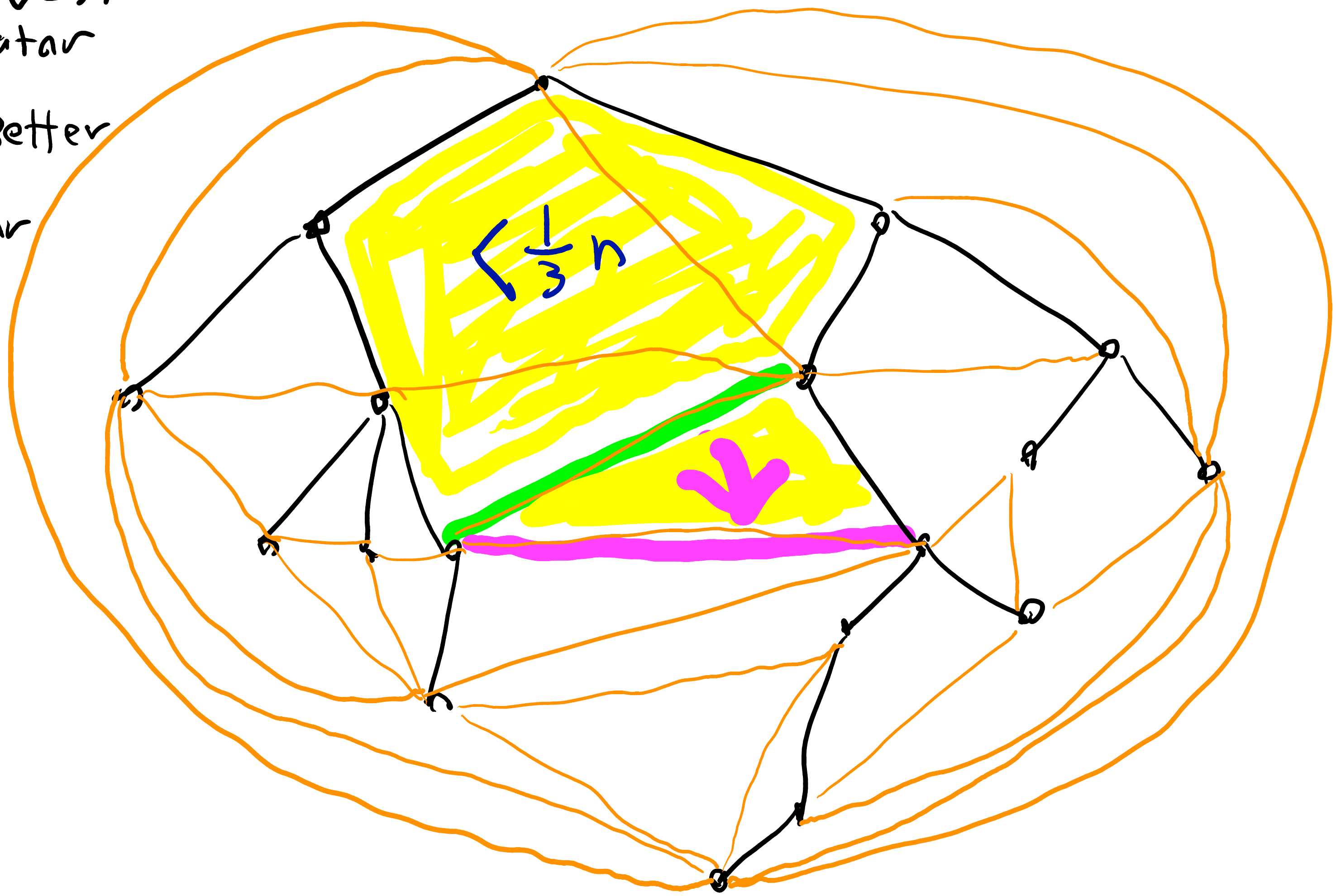


Planar Seperators for Graphs of Bounded Radius

= Best seperator

= Better seperator



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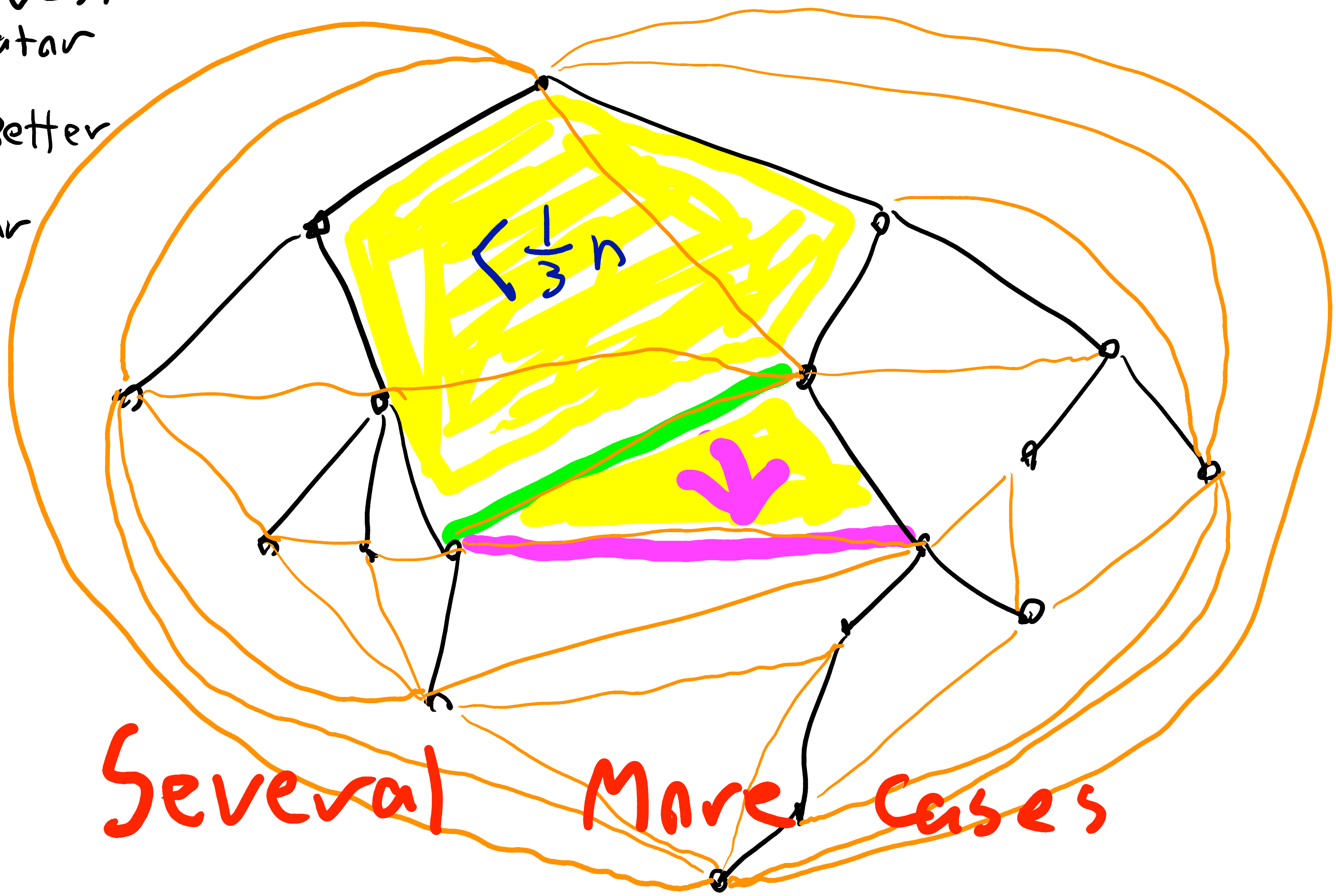
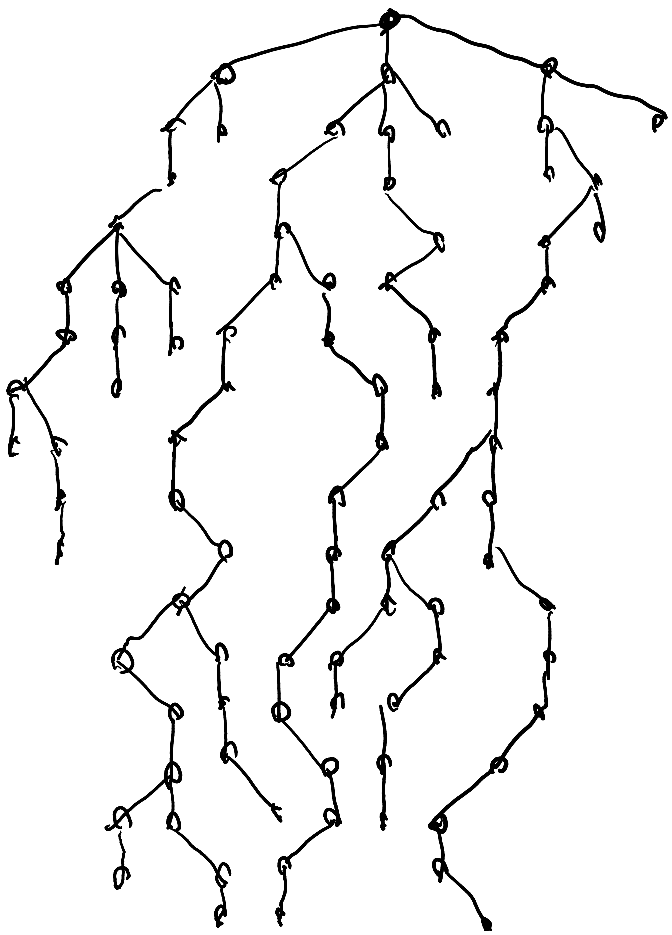


Several More Cases

Constructing The Seperator



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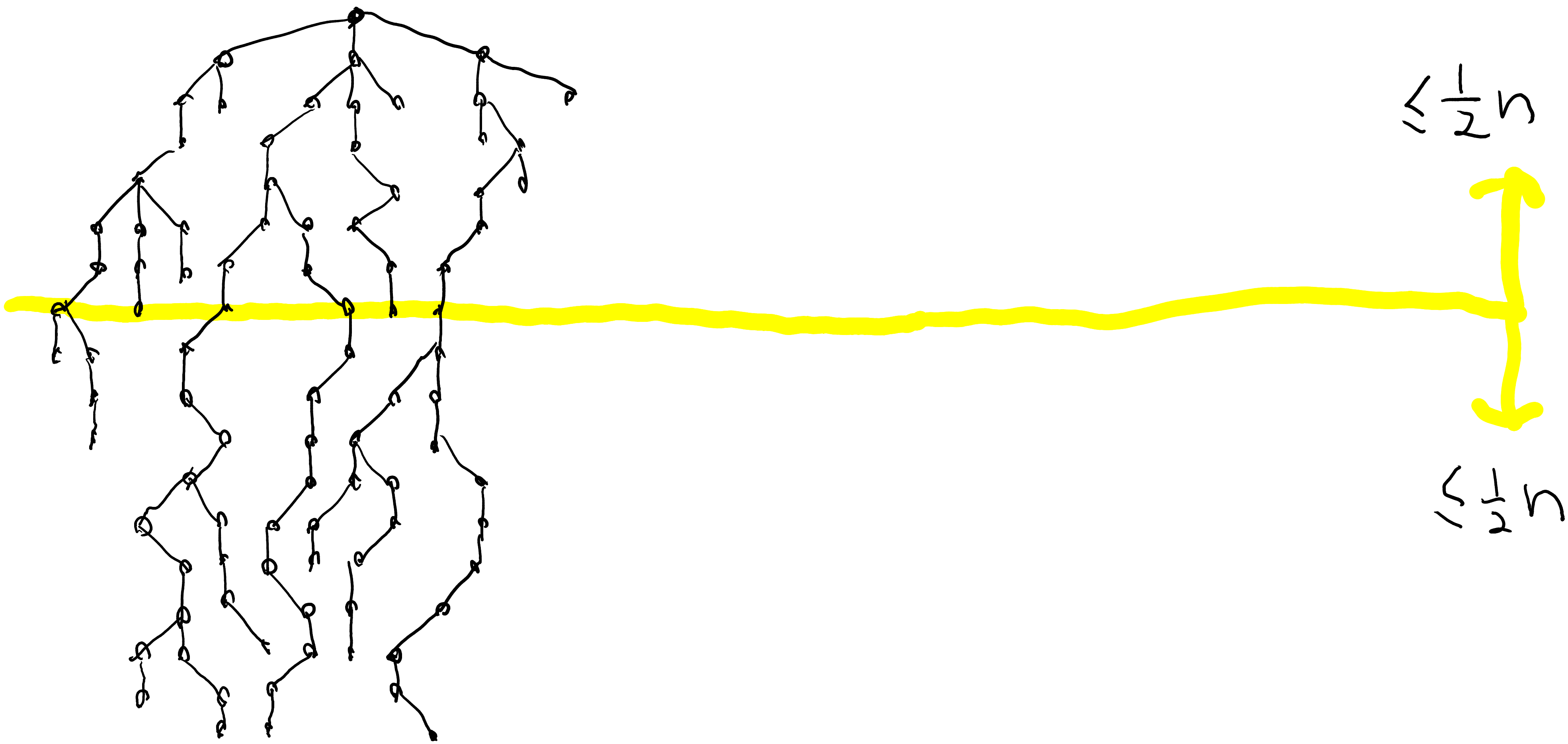

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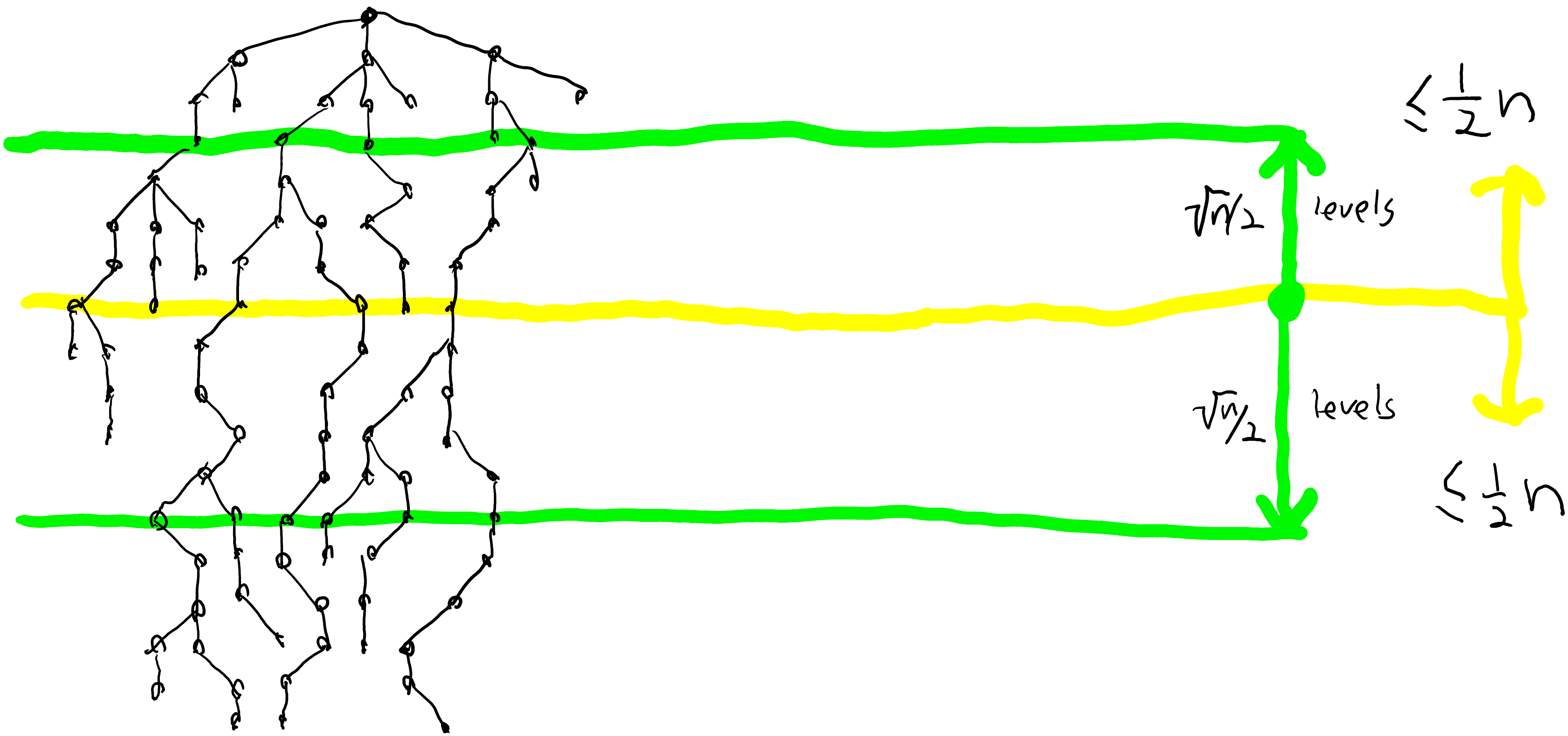

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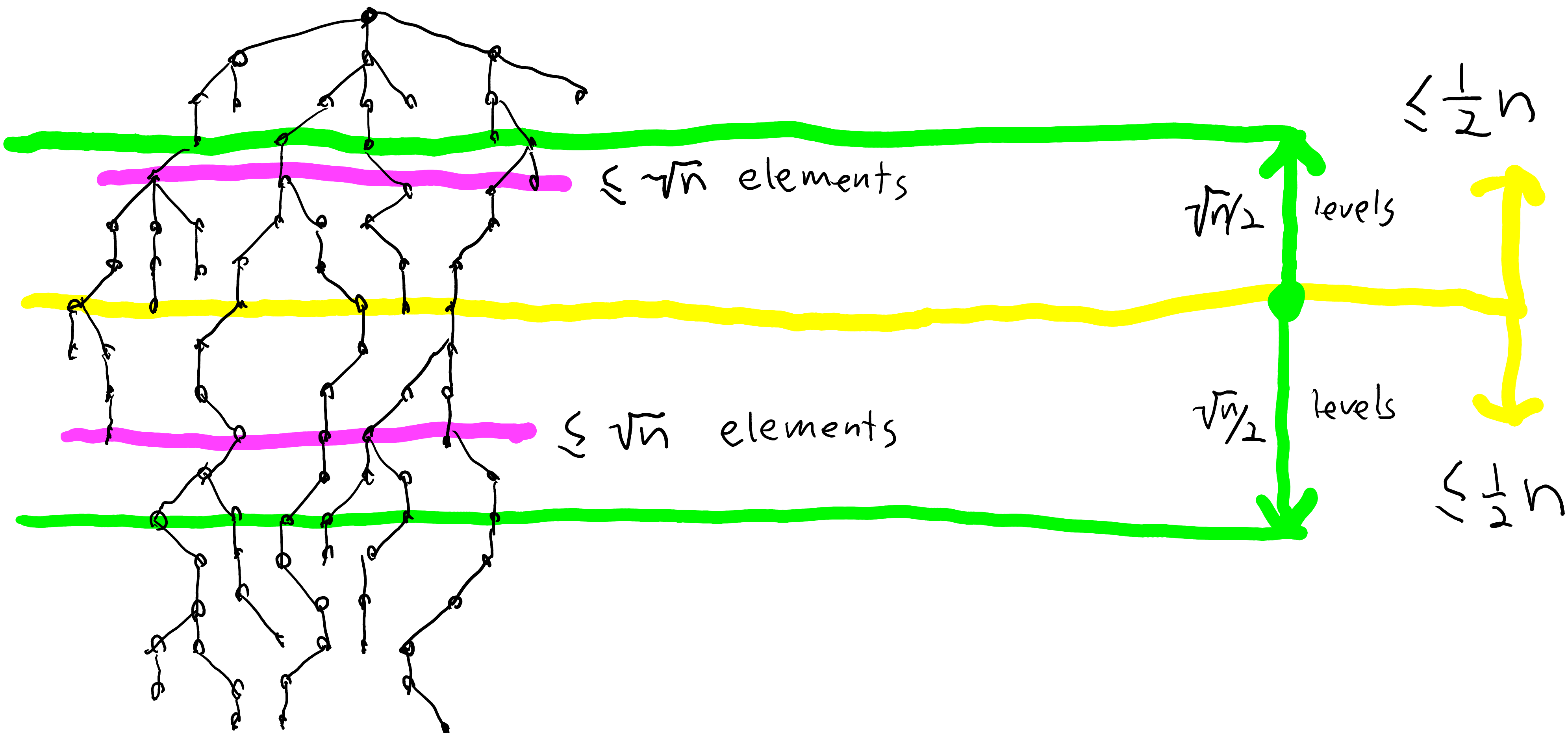

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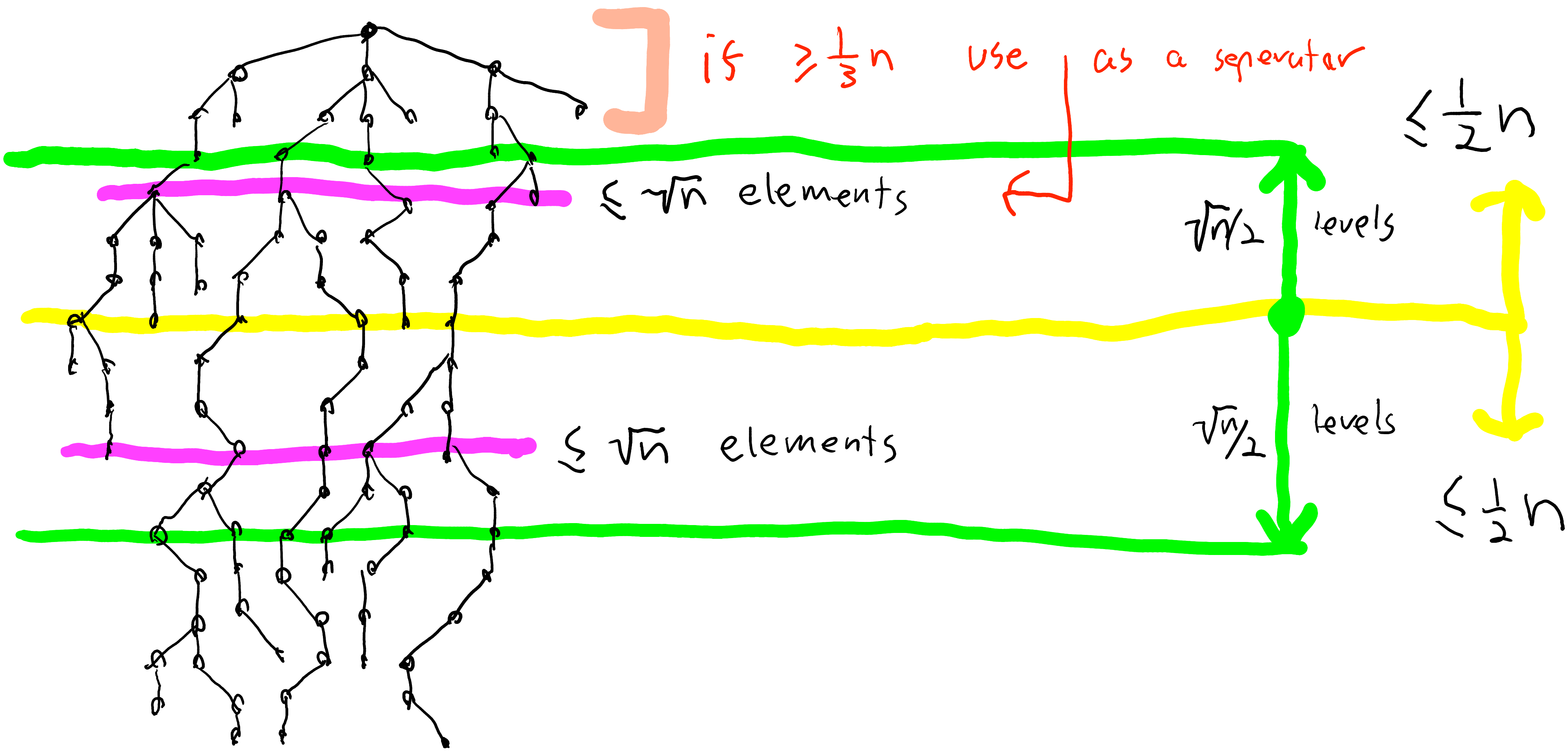

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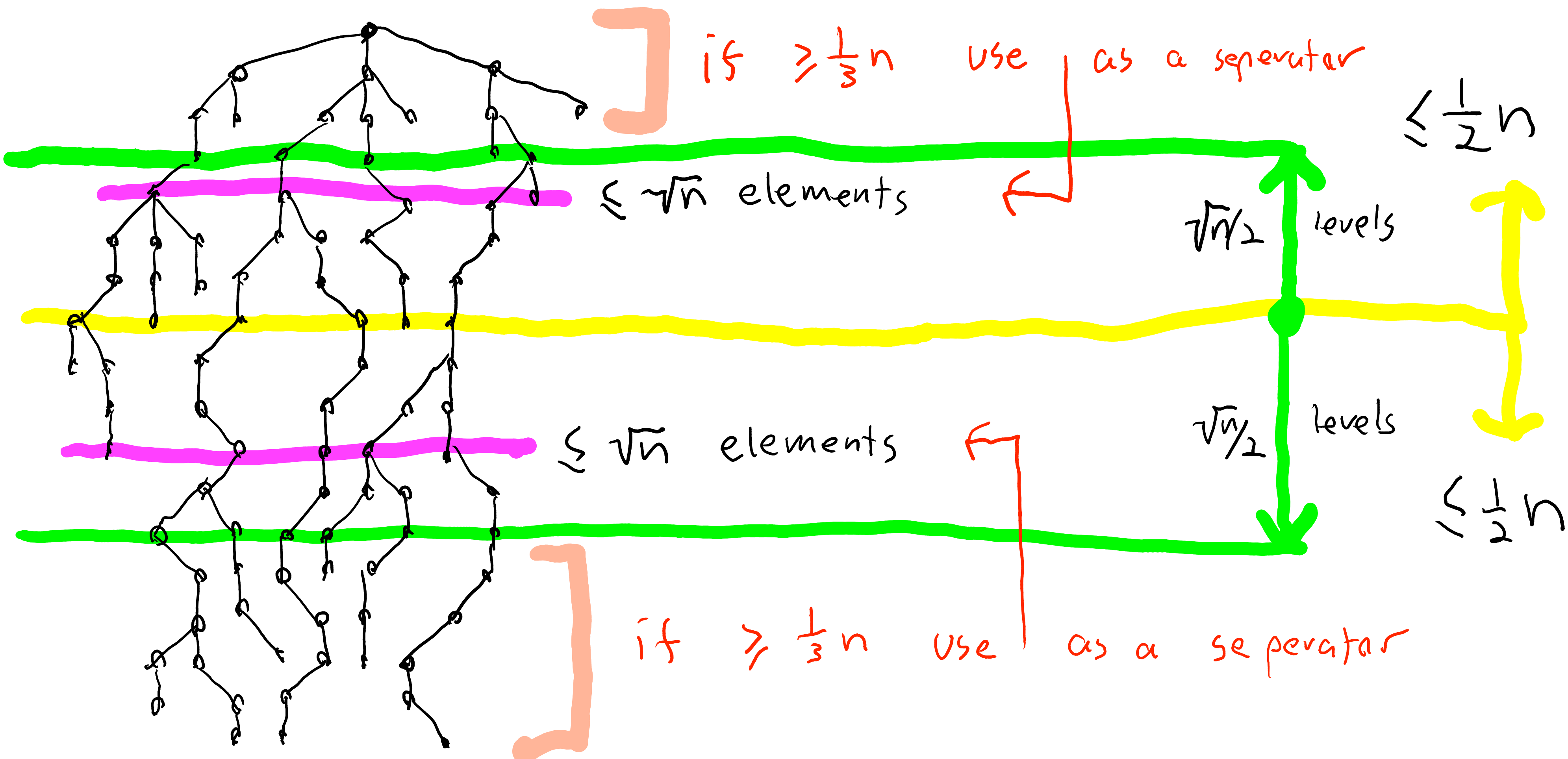

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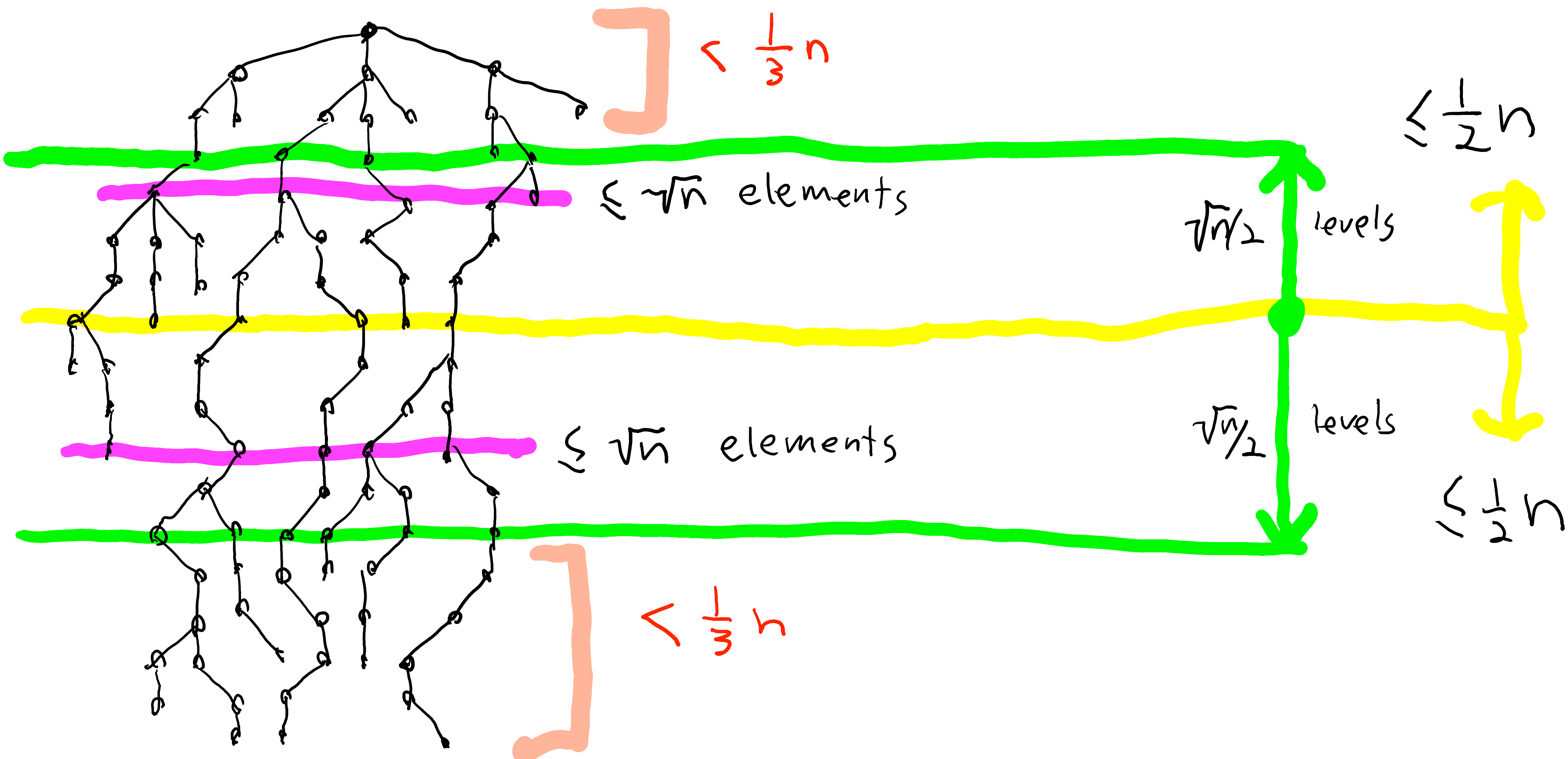

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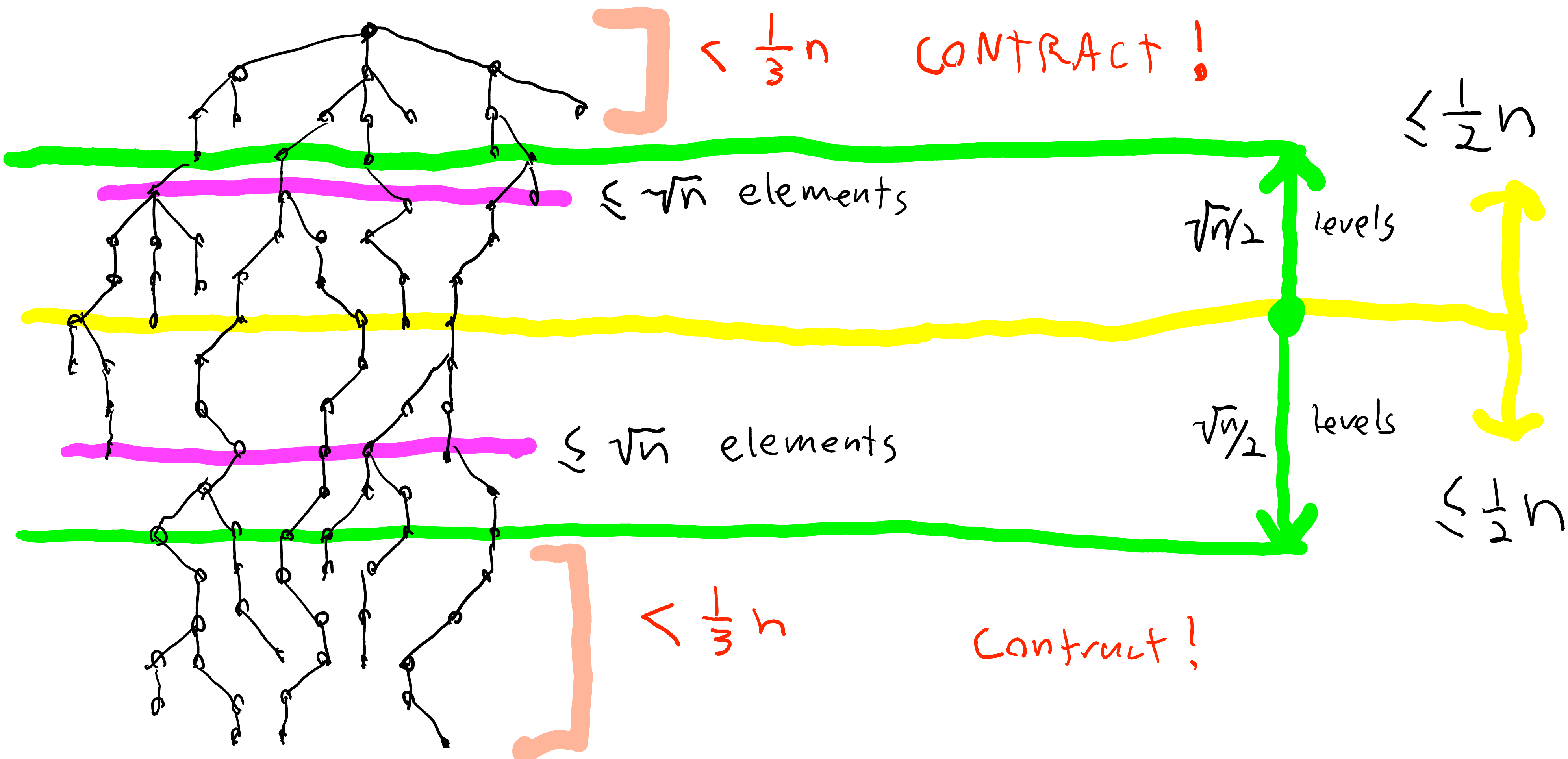

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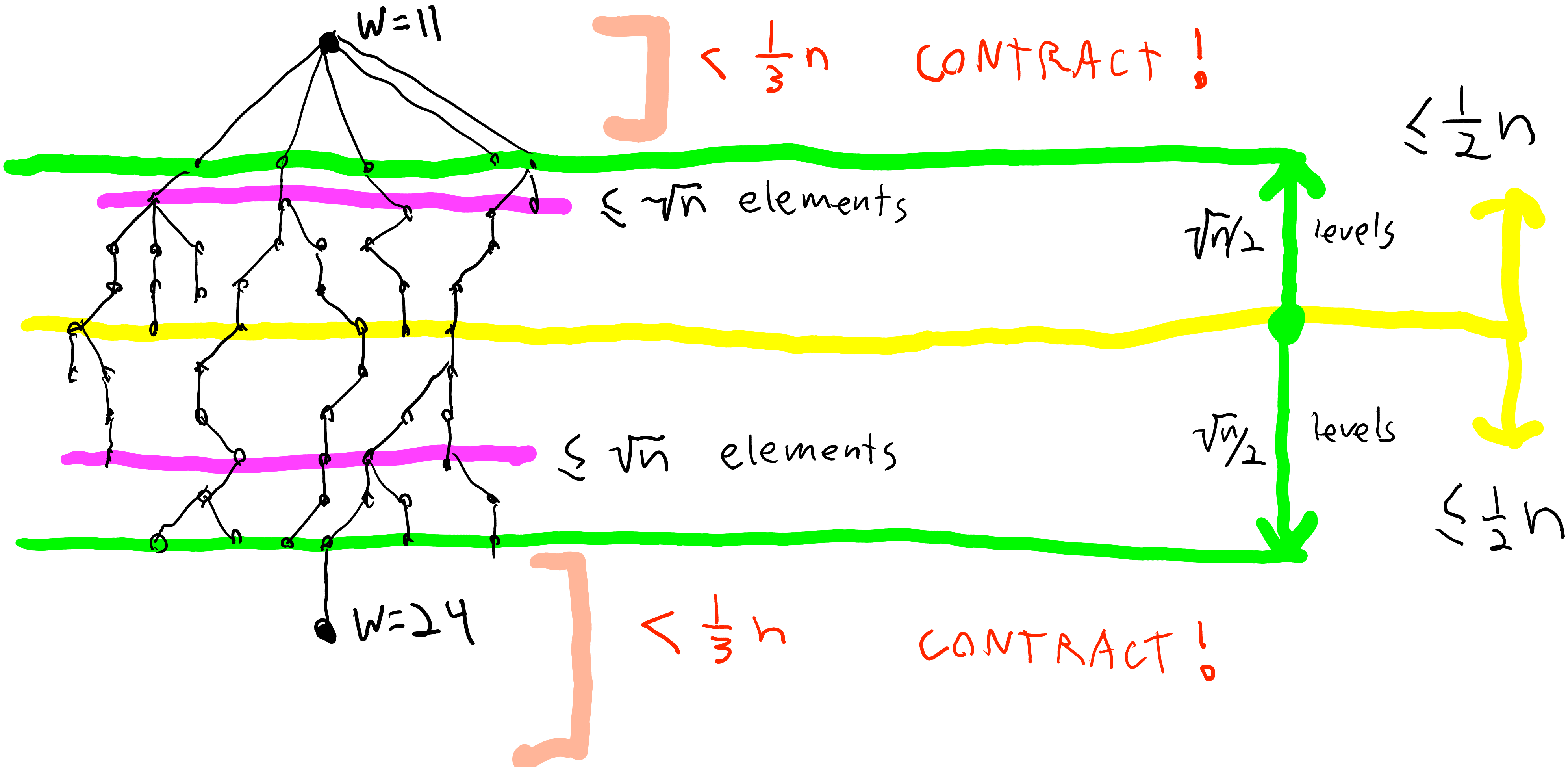

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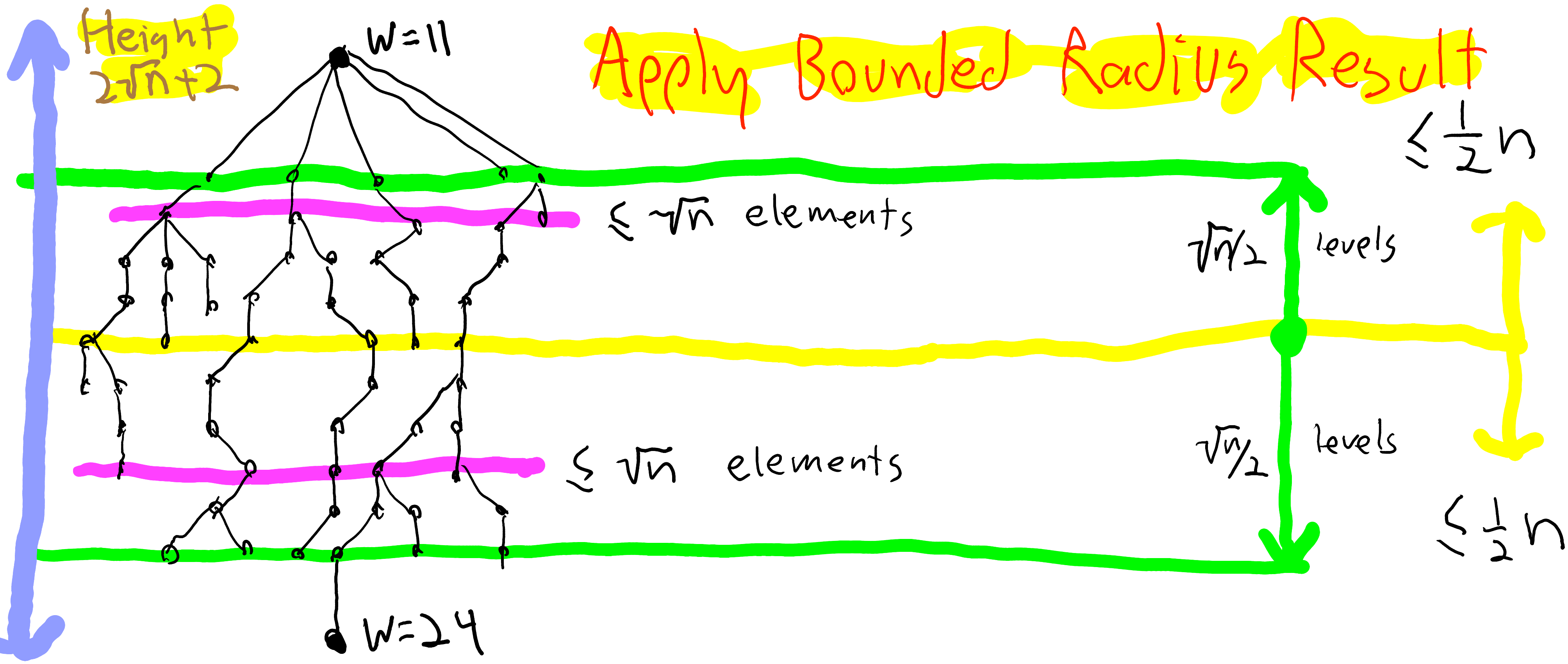
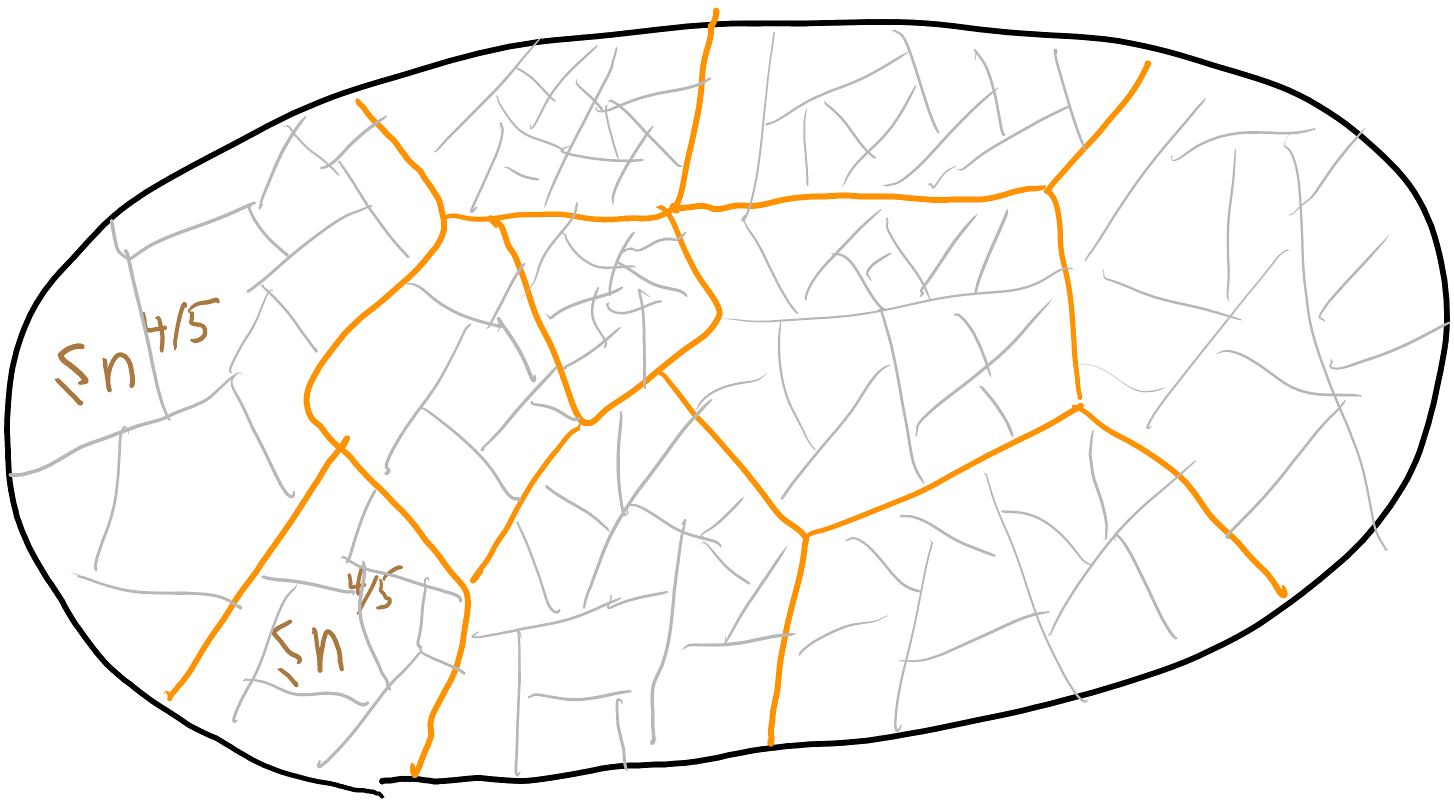


General Seperators

Seperator size	Component size
$O(\sqrt{n})$	$O(n)$
$O\left(\sqrt{\frac{n}{\epsilon}}\right)$	$O(n\epsilon)$
$O(n^{c})$	$O(n^{2-2c})$
$O(n^{2/3})$	$O(n^{2/3})$
$O(n^{3/5})$	$O(n^{4/5})$

Point Location With Seperators

$\leq n^{3/5}$



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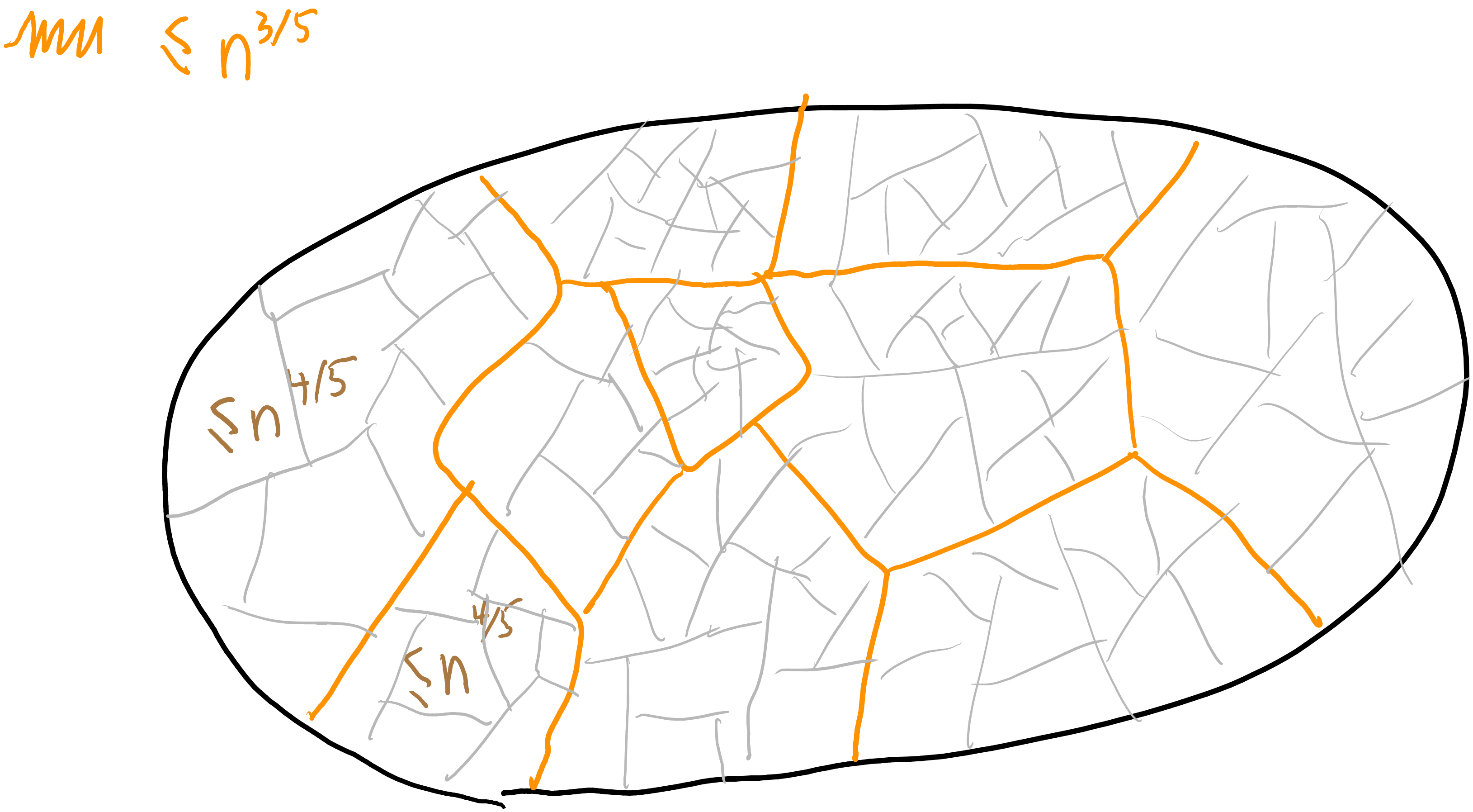

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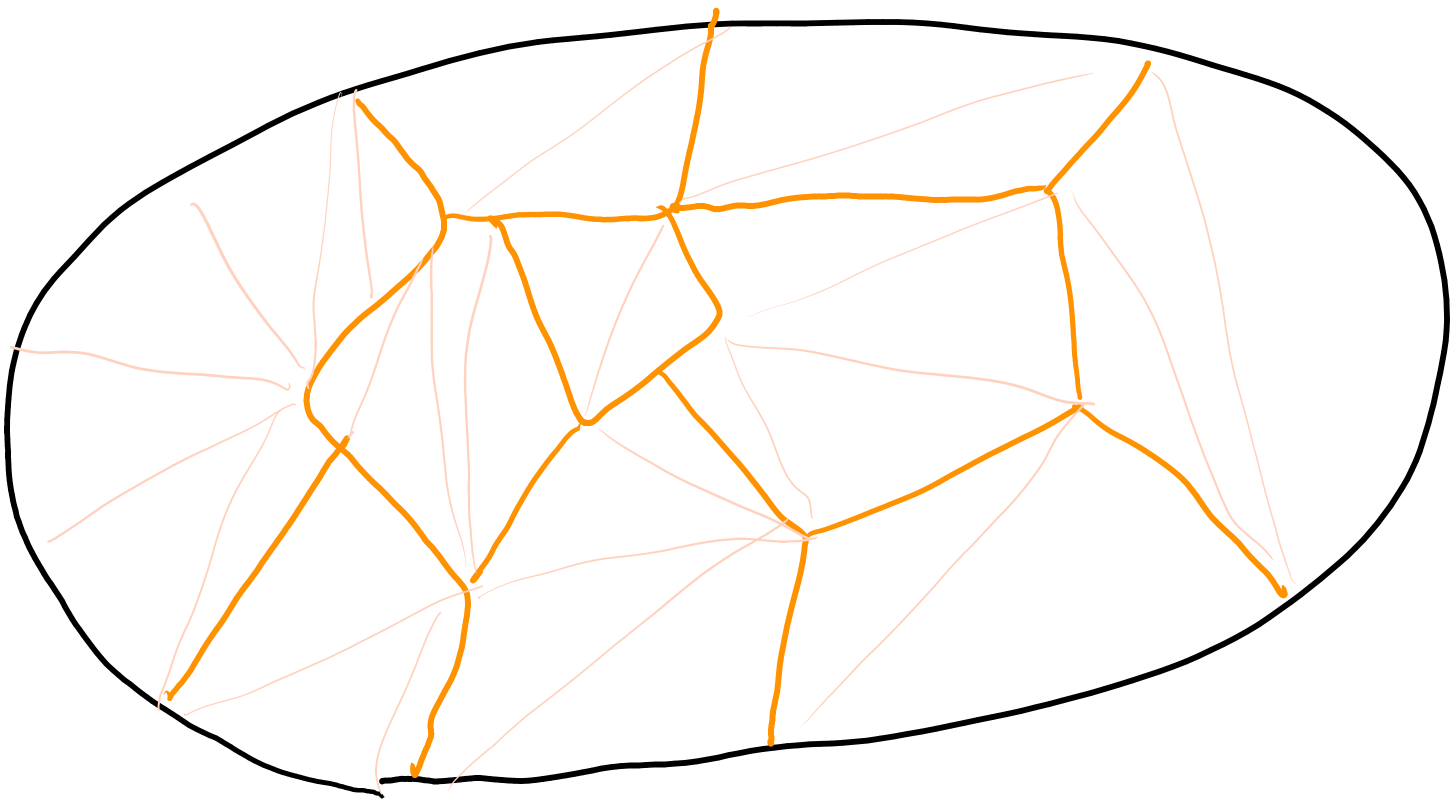

- Remove non-seperator edges, triangulate
- Size of this triangulation is $O(n^{3/5})$
- Build $O(\log n)$-query $O(n^{4/5})$ space pt location structure on this "rough" subdivision (To Be Described)

Time / Space

$T(n) = O(\log n) + T(n^{4/5})$ $\quad$ $T(n) = O(\log n)$

$S(n) = O(n^{4/5}) + n^{1/5} S(n^{4/5})$ $\quad$ $S(n) = O(n)$

* Assumes perfecty even split.
With some more complication
you get the real result

Need $O(\log n)$ structure that uses $O(n^{4/5})$ space on input of $O(n^{3/5})$

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Lemma 1: There is a $O(n^2)$ space $O(\log n)$-query structure

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Lemma 1: There is a $O(n^2)$ space $O(\log n)$-query structure

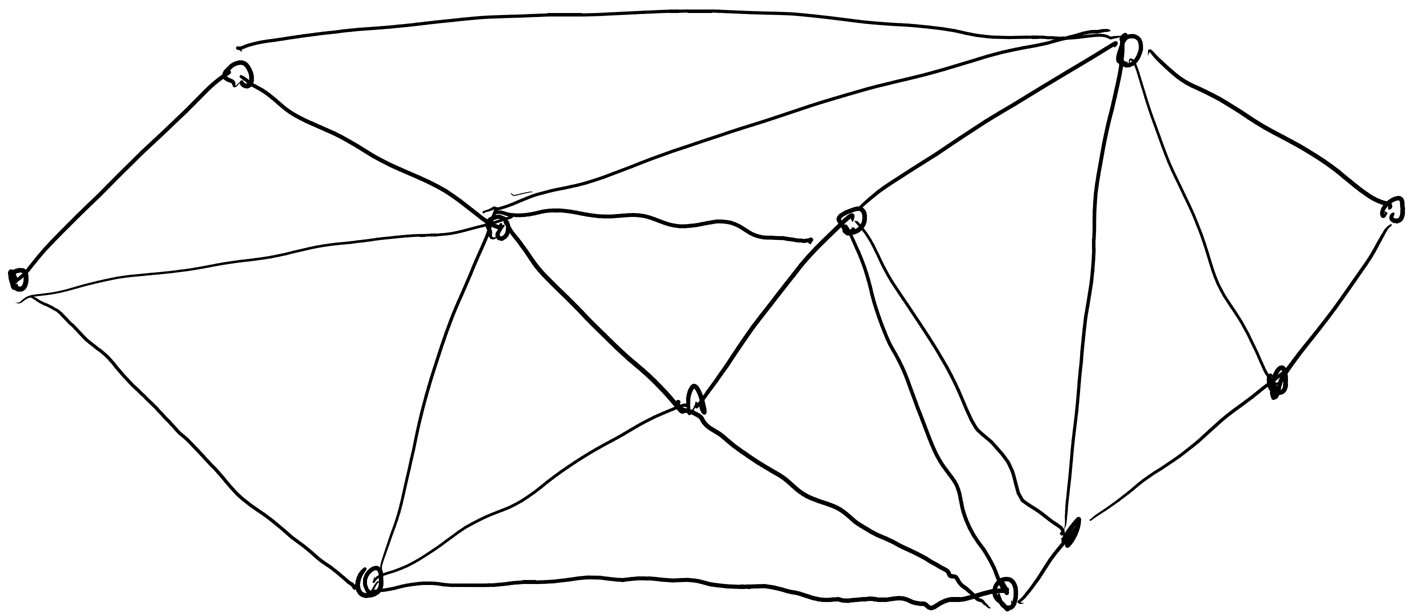
Lemma 2: $O(n^{1+\epsilon})$ space $O(\log n)$-query

$\longrightarrow$ $O(n^{1+\frac{2}{3}\epsilon})$ space $O(\log n)$-query

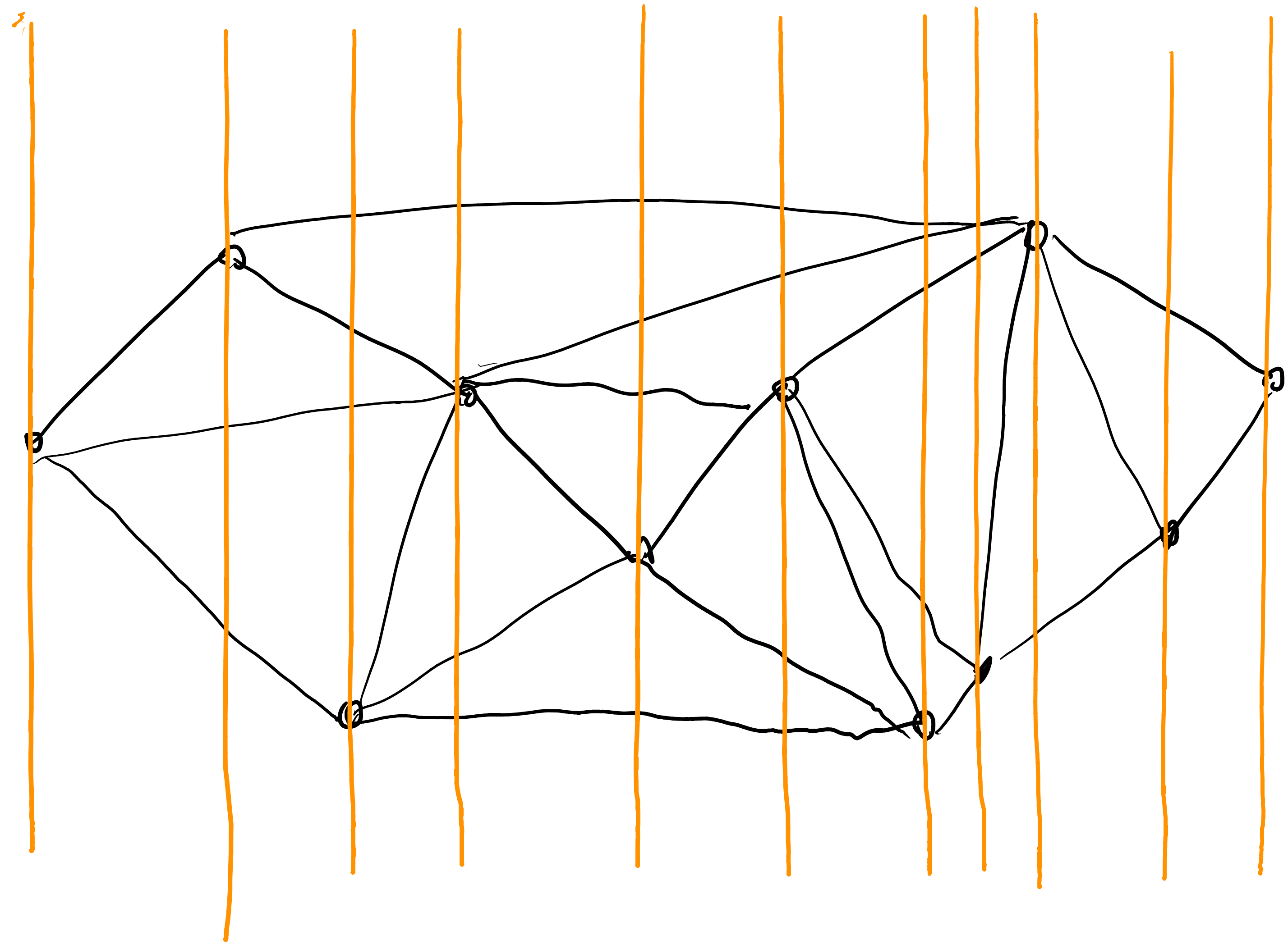
Corollary: For any ϵ there is a

$O(n^{1+\epsilon})$ space $O(\log n)$-query

L1: $O(n^2)$ space $O(\log n)$ Query



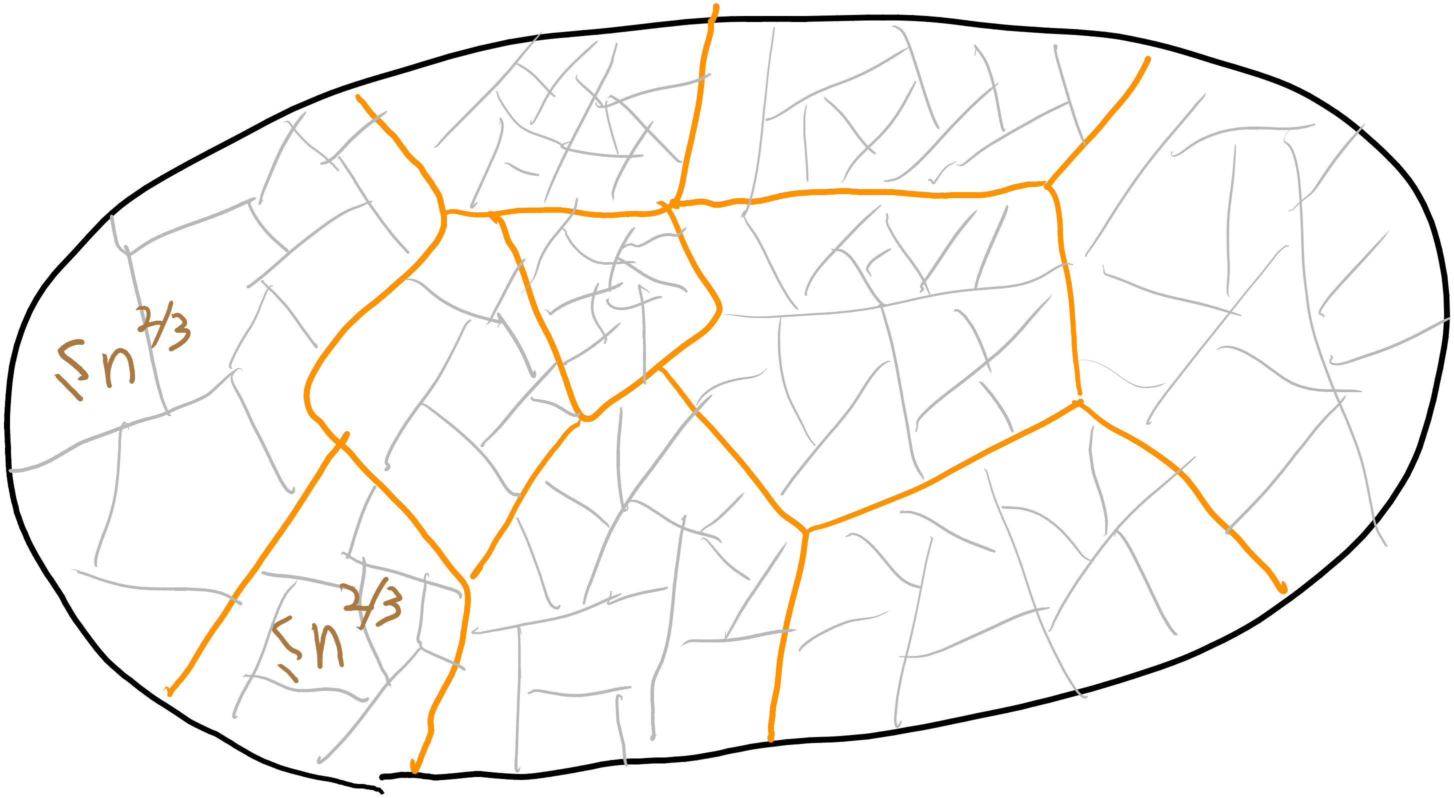
L1: $O(n^2)$ space $O(\log n)$ Query [Shamos STOC 75]



- Store each slab in a BST
- Use a BST to determine which slab you are in

L2: $\underbrace{O(\log n)\ Q\ \ O(n^{1+\epsilon})}_{*}$ sp $\longrightarrow$ $O(\log n)\ Q\ \ O(n^{1+\frac{2}{3}\epsilon})$ sp

$\leq n^{2/3}$



Use * to store each seperated cell and the triangulation of the seperator

$$\text{space} \approx n^{1/3} \cdot (n^{2/3})^{1+\epsilon} = n^{1+\frac{2}{3}\epsilon}$$

Query = 2 $O(\log n)$ queries.

Summary

- $O(n)$ space
- $O(\log n)$ query
- $O(n \log n)$ construction time

- Constants terrible